

NEUROPROBABILITY - THE JANUS PROBABILITY THIRD FACE IN COURT

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ABSTRACT

Usually the probability theory is approached from a purely mathematical viewpoint or, not entirely in alternative, from a philosophical perspective. If one confines to the mathematical perspective, probability must be seen as a primitive concept, in a Kolmogorov sense. To discuss the content of the concept, a more comprehensive framework of the Knowledge Theory is needed. In this paper it is intended to present another approach based on the concepts that are typical of Neuroeconomics, that go beyond the rationality either quantitative or qualitative. This may be described simply by the word "Neuroprobability". Reflections in the notion of probability, which began with questions related to hazard games problems, allowed a much more simplified approach in many problems that arise every day. But the emergence of different approaches, different schools, and the debate around it suggests that different scenarios allow different mind moves. The epistemological approach is supported following the subjective notion of probability, but not entirely denying that in certain phenomena another one may be adopted. And often some decisions about random events are taken in the form of pure reactions, not supported for any kind of reason, as it happens for example in Neuroeconomics, giving rise to what we may call a different concept of probability, the Neuroprobability.

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[I] INTRODUCTION

In this work it is intended to analyze the different forms assumed by the decision process, about random events, in what concerns mainly the one practiced by the judges in court. Indeed, courts necessarily have to make decisions under uncertainty, consequence of their own nature. They have to produce decisions related to the past events that must be evaluated, but sometimes they are not, in every presented case. Whenever it is mentioned traces or evidence, either scientifically or not, it is understood incompleteness of knowledge, therefore one has to assess uncertainty. A trace is a sign. To be able to say something more, one has to determine its importance, or weight, for each case, using knowledge and considering the hypotheses under evaluation.

The increasing development of the techniques and the methodologies also increase the need to properly evaluate the presented information. Thus, along with a qualitative assessment inevitably arises the quantitative, which reflects the uncertainty evaluation, in the case of the forensic context.

The probability theory can be approached from a purely mathematical viewpoint or, in another view, from a philosophical perspective. If one confines to the mathematical perspective, "probability" must be seen as a primitive concept, in a Kolmogorov sense. To discuss the content of the concept it is necessary a more comprehensive framework of the Knowledge Theory. The first significant developments in the mathematical theory of probability are dated on the second half

of the 17th century undertaken by Leibnitz or Locke. Nevertheless the debate enlargement either on the use of mathematical tools or on what concerns the philosophical sense was established in the beginning of the 20th century.

The development of the mathematical probability theory shows that, from Fermat and Pascal to Laplace, the engine of growth set in the hazard games problems, although there were attempts to apply it, by some mathematicians, in other areas specially driven for the first social statistics data collections. There were also attempts aspiring to apply the mathematical approach to problems that intended to estimate the probability of an accused individual being guilty, based on the presented evidence. The earliest use of probabilistic arguments in legal decisions, even in an incipient form, seems to have occurred more than 18 centuries ago in Babylonia and Israel with the Jewish scholars. The reflections related to the notion of probability, which began with questions related to hazard games problems, allowed a much more simplified approach in many problems that arise every day. But the emergence of different approaches, different schools, and the debate around it suggests that different scenarios allow different mind moves. Here it is supported an epistemological approach following the subjective notion of probability, but not entirely rejecting that to certain phenomena may be adopted another approach. It is assumed a conciliatory attitude as opposed to leave unanswered many problems. To consider probability Janus faced appears to be necessary in

theoretical terms as in its interaction with the practical applications.

A third alternative is to consider what may be called subjectivity beyond rationality. One aspect of this alternative is that facing the same evidences and probabilities two different judges do not decide necessarily in the same way. The other is that although facing the evidence and the respective evaluation, the decision of a judge may differ in accordance with different stimulus experienced recently or older, even if the written decision is based in the evidence and the respective evaluation. One example of this kind of stimulus are the so called convictions, sometimes passions, the most of the times unexplainable, of the judges and in the same sense of the members of the jury, in jury trials.

It is this mode of dealing with probability that here is called Neuroprobability, the third face of Janus, maybe not very correctly but that emphasizes a different behavior, in face of the same situation, from those described by the two faces of Janus. Similar situations are studied in the Neuroeconomics context where, for instance, acquisition of goods is determined not necessarily thinking in concepts like price, utility, evaluation, ... but due to any stimulus supplied by the experience of the buyer: the advertising, a pleasant experience, ...

[II] FOUNDATIONS OF PROBABILITY

The Probability Theory is a powerful tool to model the human, rational, behavior in this context. So, it is important to present its foundations. So considering a transcription in common language of Kolmogorov [1] construction it is usual to consider the probability space (Ω, \mathcal{A}, P) in which:

- Ω is a fundamental non empty space - generally named outcomes space - composed by elementary events $w_i \in \Omega$;
- \mathcal{A} is a non empty family of Ω subsets, closed for the usual Boolean operations. These sets $\{A \in \mathcal{A}\}$ are entities for which it is possible to associate a non negative real number, i. e., a probability;
- P is an additive function which domain is \mathcal{A} , such as:
If $A \cap B = \emptyset$ then $P(A \cup B) = P(A) + P(B)$.

Kolmogorov [1] also generalized the additive property for non finite spaces (Ω) provided with non finite algebras (\mathcal{A}) , but contrarily to what had been said he did not advance from the structure of algebra to a structure of σ -algebra. To force a structure \mathcal{A} of subsets of Ω to be closed for operations of sets in non finite number gives rise to some small monstrosities which the observer is not able to identify.

One may have some prevention to the generalization of the additive property for non finite spaces provided with non finite

algebras. The most common attitude consists in imposing to \mathcal{A} a structure of σ -algebra and to substitute the last Kolmogorov axiom with the generalized additivity. In fact, this was not followed by Kolmogorov. He added a sixth axiom:

Axiom of continuity:

Considering $A_1 \supset A_2 \supset \dots A_n \supset \dots$ and $\bigcap_n A_n = \emptyset$ then $\lim_n P(A_n) = 0$. ■

He also added the theorem:

Theorem

If A_1, \dots, A_n, \dots and $A \in \mathcal{A}$ and $A_i \cap A_j = \emptyset, i \neq j$ with

$$A = \bigcup_{i=1}^{\infty} A_i \text{ then } P(A) = \sum_{i=1}^{\infty} P(A_i). \blacksquare$$

Which demonstration results from the acceptance of the axiom of continuity.

The numerable additivity raises some objections within the Subjectivists (see Kyburg and Smokler [2]). In fact, Epistemological theories see the probability as a state of mental uncertainty about an event. These theories can be divided into logical and subjectivists theories. Logical theories suppose the existence of a single rational degree of uncertainty about the event. However, the problem is that it is not known yet. The subjectivist, but rational, interpretation has become more popular in the last years. Subjectivists regard probability as a degree of reasonable belief in a certain event, from an individual viewpoint. Therefore probability is a numeric subjective measure of a particular person according his/her degree of belief, as long as it is 'coherent' - avoiding the Dutch book.

Following Savage, see [3], an economist that used mathematical tools to model the Economic behavior, "It may seem peculiar to insist on σ -algebras as opposed to finitely additive algebras even in a context where finitely additive measures are the central object, but countable unions do seem to be essential to some of the theorems...

So much of the modern mathematical theory of probability depends on the assumption that the probability measures at hand are countably additive that one is strongly tempted to assume countable additivity or its logical equivalent, as a postulate. But I am inclined to agree with de Finetti and Koopman that, however convenient countable additivity may be, it, like any other assumption, ought not be listed among the postulates for a concept of personal probability unless we actually feel that its violation deserves to be called inconsistent or unreasonable.

It therefore seems better not to assume countable additivity outright as a postulate, but to recognize it as a special hypothesis yielding, where applicable, a large class of useful theorems”.

To Savage's objections one may add the de Finetti's, “No-one has given a real justification of countable additivity (other than just taking it as a “natural extension” of finite additivity); indeed, many authors do also take into account cases in which it does not hold, but they consider them separately, not as absurd, but nonetheless “pathological”, outside the “normal” theory.

Countable additivity cannot, therefore, be conceived of as a general principle which leads us safely around within the special field, and allows us to roam outside, albeit in an undirected manner, with an infinite number of choices. On the contrary, it is like a good-luck charm which works inside the field, but which, on stepping outside, becomes an evil geni, leading us into a labyrinth with no way out”, de Finetti [4].

These objections are very close within the careful thinking line in Kolmogorov approach that is not taking the σ additivity as an axiom - generalized of finite additivity - but instead consider that it works under certain conditions: axiom of continuity and circumstantial “closeness” - not structural - for a certain

numerable union of events - $A = \bigcup_{i=1}^{\infty} A_i \in A$.

[III] CONDITIONAL PROBABILITY: BAYES THEOREM

Taking into consideration the comments above, one may follow considering A_1, A_2, \dots, A_m a finite or non finite partition of Ω with

$$P(A_i) > 0, A_i \cap A_j = \emptyset, i \neq j, \bigcup_i A_i = \Omega.$$

Given any other event B , with $P(B) > 0$, it is easy to see the decomposition of B as a union of disjoint sets

$$B = \bigcup_i (A_i \cap B).$$

Consequently, assuming for the present case the additivity of the function P and the definition of conditional probability, then

$$P(B) = \sum_i P(A_i \cap B) = \sum_i P(B|A_i)P(A_i)$$

therefore

$$P(A_i \cap B) = P(B|A_i)P(A_i) = P(A_i|B)P(B)$$

and settling $P(A_i|B)$ it is obtained:

Bayes' Theorem (also called Bayes' Law):

$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{P(B)} = \frac{P(B|A_i)P(A_i)}{\sum_i P(B|A_i)P(A_i)}.$$

Note:

- Considering

$A_i, i = 1, 2, \dots, m$, as m hypotheses, $H_i, i = 1, 2, \dots, m$, and B as data, being I the initial information, Jaynes [5] presents the Bayes's Theorem in a different way (see Andrade [6])

$$P(H_i|Data, I) = \frac{P(Data|H_i, I)P(H_i, I)}{\sum_i P(Data|H_i, I)P(H_i, I)}.$$

[IV] A COMMON PROBLEM

In each case the judge, or jury, has, necessarily, to make a decision - *Non Liquet* principle. Although it is a decision problem, it cannot be understood, studied and solved by the methodologies presented in the Decision Theory.

This context, in which there is always a decision, it is not adequate to use the “tools” of the Decision Theory, which is based on an utilitarian approach for the different possibilities - although there are also followers of the utilitarian theory among the Law area theorists.

On this concern, one can say that there is an agreement in the Law area: The task that the judge has before him is the following: to find a decision, solution, founded by the law, Engisch [7]. Perelman also states that the law as actually works is essentially a decision problem: the legislature must decide which laws are mandatory in an organized community; the judge must decide what is right in each situation brought to his trial, Perelman [8]. And also Larenz [9]: the judge's task is to determine legally factual situations that have occurred, and that there were only imagined.

What seems not to reach a consensus is that lawyers and statisticians may in some issues, to have to deal with similar problems. Of course, it is recognized that Statistics and Law are autonomous and deal with specific problems. In fact, *prima facie*, it seems that those sciences have little or nothing in common. Statistics immediately suggests a quantitative relationship with the phenomenon under approach, whereas Law, using argumentation, the laws and the decisions, which is

taken following the contours of the laws and the consciousness of the "decider", presents a more qualitative treatment of the topics of interest. Even a layman in the field of Law, accepts that the disciplines are far more than that. Some might even admit that there will be eventually identical points between them. Following Dawid, "although the concerns of Statistics and the Law might seem to have little to do with one other, they do share some fundamental common interests, such as interpretation of evidence, hypothesis testing, and decision making under uncertainty", Dawid [10].

In what concerns those who operate in Law, whether in practice or theoretically, e.g. judges, lawyers, there is an almost unanimous shared idea that mathematics, in a general sense or, more specifically, some branches of Mathematics and Law are not related disciplines or even conciliable. "It is not a mathematical formula ...", "It cannot be translated into a number ...". That is why the reasoning of a judge is dialectical opposite to the reasoning of mathematicians, who always walk in one direction, from premises to conclusions. (...) The reasons given by judges would be arguments that are not coercive, as in a mathematical proof, according Perelman [8]. These are examples of beliefs that will be encountered when seeking to inquire about sharing common interests between Statistics and Law, from latter's representatives. In fact, it is not intended to provide an algorithm or sensational formula as a solution, but rather to look for common elements, realizing that the problems that both deal with are, many times and in many ways identical. Although the approaches are different, broadly speaking their common interest is dealing with evidence interpretation.

The question that the judge has to answer is: After the case being presented what is the posterior probability of the facts based on evidence presented? The judge must evaluate the evidence presented and the arguments of the different parts, defence and prosecution, arguing about the hypotheses in dispute. Based on the exposed case, and using a reflective analysis regarding the situation under appreciation, and supported, sometimes also in their experience, the judge reaches a conviction and decides. As it is known the judge mission is to administer justice, and the whole decisions must be justified and grounded, which allows everybody to understand the reasons for either decision. It is important to mention that to accomplish a conviction the judge makes use of legal and not so legal reasons.

"The speakers who addressed the judge can rely on all the rules of law and procedures available to the process and the judge cannot refuse them without being guilty of a violation of the law. Moreover, it is according to those rules that the judge must support his sentence, so as to obtain the consent of their peers, their superiors and the opinion of jurists, on the fact that has issued a decision according to the Law. It is known that, along with rules of law that anyone seeks to challenge, or to interpret its own way, the whole Law system contains a sufficient

number of uncertainty elements, which gives the judge enough freedom, and depends on both the inner conviction of Judge regarding the establishment of the facts, that the judges' personality always plays a role, sometimes limited, but often also decisive in the process and its result", Perelman [8].

As Perelman stated in Law one is faced with the dialectic of the reason and the will versus the reality and the value, being the reason and the reality the objective part, the one that the judge must take into account and should be leaning, providing the will and the value subjective part which depends, ultimately, of the judge's decision, Perelman [8]. "Acknowledging the power of judge's decision that manifests itself through the subjective part, it should be noted that this power is not arbitrary, i.e. it is not an optional or despotic power which the judge can use without control, since all decisions must be reasoned. Whatever functions can the irrational sources of the discovery of the judicial statements or the decision perform, the judge confronted in his position (function) and conscience, only can feel justified when his decision may also be based on the Law, which means being derived from it", Engisch [7].

Thus, it is possible to agree that the Law operates with decision making, which is not contrary to reason whenever justified by an argument that is recognized. It is true that conclusions of the arguments are not compelling, and so to agree with all convictions.

The argument based on the evidence presented may allow influencing the direction of the decision, supported by the most convincing arguments, but it is not the only way in concrete situations. Other kind of "reasons", may be called "non-reasons", as the stimulus described above may influence either the initial conviction of the judge or the final appreciation after joining that conviction with the appreciation of the evidence. Cultural aspects, prejudices, education, convictions may be joined or even replace the computation of probabilities and the legal aspects in the building of the decision. And even the appreciation of a number may differ from a judge to one another according to those factors.

This mechanism of belief creation may be interpreted as the replacement of the probability computation by the consideration of a probability built through neuro-stimulus: the so called Neuroprobability.

[V] FROM TWO TO THREE JANUS FACES

The philosophical meaning of the probability concept has originated very different ideas. Consequently in an initial moment, four main currents of interpretation appeared. Following Gillies [11], these interpretations can be summarized as follows:

- Logic Theory which identifies probability with a reasonable degree of uncertainty. It considers that before the same evidence all rational human beings have the same belief in a certain hypothesis;
- Subjective Theory which identifies probability with a degree of belief that each individual has in a certain hypothesis. It is allowed the difference of opinion between different individuals;
- Frequency theory that defines the probability as the "limit" of proportion of successes in a sequence of experiences;
- Propensity Theory to which the probability is an inherent propensity within a set of repeatable conditions – actual or virtual – (Among those who advocate logical theory of probability was John Maynard Keynes who stressed his more philosophical aspect, for whom the probability is defined as the degree of partial causality (*probability is the degree of partial entailment*). Ramsey and de Finetti, independently, were the forerunners of the ideas concerning the subjective theory of probability, during the 1920s and beyond. The frequentist theory initially followed by Ellis and Venn was later developed by Reichenbach and von Mises two thinkers closely linked to the Vienna Circle. The propensity theory was introduced by Karl Popper in 1957 and latter developed and explained in his works in 1983 and 1990).

During the historical discussion different approaches of the concept have risen, however a systematic classification has not been consolidated. In 1983 Murteira [12] has noticed that compared to the antagonism between the Classical and Bayesian Box attempt through a dualistic theory of statistical inference to reconcile them, Murteira [12], for whom the doctrines more than competing, are complementary. Box "ecumenism" is reflected in a division of the work: to Frequentists the critical (the model is adequate?), to Bayesians the estimation (if the model is adequate then estimate the parameters!), Murteira [12]. In 1994 Gillies [11] proposes to divide the interpretations of probability in Objective and Epistemological. The objective interpretations consider probability as a property of material world, where human knowledge through observation, will quantify the uncertainty, i.e., the uncertainty is in nature. The epistemological interpretations conceive probability as related to the degree of belief or knowledge of human beings. According to this perspective the probability measures the degree of knowledge or belief of each individual, moving the uncertainty into the perspective observer/ phenomenon.

These two conceptions of probability describe the rational approaches to random events appreciation. The emphasis intended here is on what is beyond this rationality even when people thinks that it is acting rationally. In fact, the ambience –

time, local, mental, ... - influence the behavior of anyone not necessarily in what it called a rational mode.

In the Roman tradition Janus was the god who gave his name to January, god of the beginnings had two sides in its representation - perhaps one looking to the past and the other looking to the future. Since mid-nineteenth century, with Poisson, Cournot and Ellis, it is mentioned the two sides of probability, Hacking in 1975 calls it the two faces of Janus: ... probability... is Janus-faced. On the one side it is statistical, concerning itself with stochastic laws of chance processes. On the other side it is epistemological, dedicated to assessing reasonable degrees of belief in propositions quite devoid of statistical background, see Gillies [11] and also Andrade and Ferreira [13].

It is following this line why it is proposed the name "third face of Janus" to describe the interpretation and the evaluation of probabilities subject to neuro-stimulus, the Neuroprobability, influencing the decision process.

In fact, the interpretation of probability concept is still a subject of intense debate, and even among the supporters of an approach are differences. It seems, however, that, in the essence, the distinction lies in this distinction between objective interpretation and epistemological interpretation of probability.

What has been observed is that the different approaches to uncertainty have declared these two conflicting interpretations. Beyond these interpretations and their consequent proposals of behavior it must be considered, in this context, the "third face of Janus" characterizing behavior.

Uncertainty is in nature and repetition is the mechanism used to determine it, argue the objectivist. But if that is accepted, then there are many problems left to unanswered arising every day, for not be incurring into contradiction. Uncertainty evaluation is supported on nature-observer for the epistemological approach, which does not state a kind of "prescription", but opens the perspective to subjectivity and to a certain plurality of mechanisms.

On one hand the objectivist current argues for the repeatability in what concerns probability, on the other epistemological current attempts have been made to establish some agreement, seeking for an enlargement of the concept.

The subjectivist school while rejecting the essential character of the frequentist theory, does not rejected it to be considered in a process that allows "repetition" and frequency analysis as an element of information in the process. Although this is a tolerant kind of approach it is also an agglutinating proposal, recognizing the viability of the process, frequency analysis, removing the autonomy as a current and coherent. The frequency analysis can be, among others, an element of

information, but more than one element can be considered a particular case, is only available to a limited number of cases. It can provide information in some cases therefore can be included in its evaluation.

The “repetition” is not essential for Neuroprobability. One only stimulus may be determinant in the conviction building. Although not rejecting it it is not essential. This fact differentiates definitively this probability concept from the others.

In Philosophical Theories of Probability, Gillies [11] describes the various theories and their philosophical meaning, proceeding with a proposal. Gillies [11] advocates a pluralistic view of probability, and admits adopting either of the objectivist or of the epistemological current, depending on the type of phenomenon or process under study, therefore trying to reconcile the concepts and their own daily practical decisions in the most various problems.

If one wants probability to become truly an operational tool in the most diverse areas as hazard games, physics, quantum or deterministic, or even the social sciences, it is important to reach the operationally of the concepts and their connection with specific methodologies in the different application areas, so that the purposes may be achieved. It seems appropriate to consider that certain phenomena exist *per se* regardless the observer and others exist only if observed, Why not to adopt different approaches in different situations?

The Neuroprobability cannot be considered an operational concept. One only may influence it trying to find the adequate neuro-stimulus, there having a lot of examples in the speeches of the counsellors in the final allegations.

The first reflections relating to the probability concept began with the hazard games. Thus, a more simplified approach was allowed. The emergence of different approaches, different schools, and the debate generated by them suggests that different scenarios allow for different approaches. For our part it is preferred a subjective epistemological approach, but it is not absolutely rejected that to certain phenomena it is adopted a different approach. It is admitted a conciliatory attitude in opposition to leave unanswered many problems. Reaffirming the probability two Janus faces, it is necessary to consider them when mentioning probability, in theoretical terms and when related with the practical applications. But of course it is imperative to note that the Neuroprobability is always present, independent of our will. So the consideration of Janus third face.

[VI] DISCUSSION: WHICH FACE TO CHOOSE

The ever-increasing ease of communication among different areas of knowledge and the amount of problems that arise

reinforce the need to question: which probability concept to adopt? What and how to articulate application of the concept (s) to the practical question (s).

It is not indifferent to opt for one or another probability concept. Following Dawid “even without (before) one chooses an interpretation it can be considered that “probability” as a purely theoretical term, inhabiting the intellectual universe and without any direct physical counterpart”, Dawid [14], being indirect the link between theoretical probability and the physical universe. In this context, the knowledge of the phenomenon under study, supported by the convictions of the “agent”, leads the choice of which interpretation to use, in each case.

Given the diversity of problems that arise, the ambition to take advantage of the concept that allows the search for different solutions, which should be wide-ranging? Although there may be (and there always is!) a preference for an interpretation of probability, to make the concept malleable allows us, for sure, a greater number of better answers.

One can say that the core element of Statistics lies in the inference. Indeed, the observation of some data for a particular phenomenon leads in making statements and inferences about one or more unknown characteristics of the system or mechanism that caused it. And that was probably what motivated the work of John Graunt (1662) Natural and Political Observations on the Bills of Mortality, which can be considered an attempt to collect data on births and deaths and the subsequent extraction of conclusions.

Note that, since the mid-seventeenth century some mathematicians have tried to apply their theory to the available empirical evidence. However, recourse to the application of mathematical theory to study real world problems has begun in a strict context of hazard games. It took some time until it could be successfully applied to economic/social practical problems. But, the theory maturation allowed finding innumerable practical applications either in natural sciences or in social sciences.

There were already presented different notions of probability that in practical applications may be different for different particular contexts. If it is true that physical phenomena often originate a large amount of repetitive information, there may not be disregarded social phenomena that are of high interest to human activity, which by their nature do not allow repeatability. The lack of quantitative theories successful in these situations stimulates the need to introduce operational procedures for quantifying what is qualitative by nature.

In court it is preferred to follow a subjective epistemological approach. But it is not absolutely impossible that for certain phenomena a different one is adopted. Reaffirming the probability two Janus faces, it is necessary to consider them

when mentioning probability, in theoretical terms and when related to the practical applications. This is the recommended procedure in court combining the conviction of the judge, jury, with the practical, experience, knowledge, i.e. the subjective and the objective probability concepts.

The Neuroprobability as it was seen above is not a question of option but of presence. And the counsellors intuitively know it very well.

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CONFLICT OF INTERESTS

The authors declare no conflict of interests.

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