

# KERNEL BASED SPATIAL FUZZY C-MEANS FOR IMAGE SEGMENTATION

Deepthi P Hudedagaddi<sup>1\*</sup> and Balakrushna Tripathy<sup>2</sup>

<sup>1,2</sup>SCOPE, VIT University, Vellore, Tamil Nadu-632014, INDIA

## ABSTRACT

An extension of various available clustering algorithms has been serving as a solution to serve many current problems by the researchers. The Fuzzy C Means (FCM) algorithm that has been in use all these days is extremely noise sensitive. Hence it fails to provide the desired results. This was solved to an extent with the introduction of spatial fuzzy c means. This included a spatial function which was the summation of all the membership values of the neighbors of the pixel considered for study. This paper proposes a new and better modification of the spatial fuzzy c means (sFCM) by introducing kernel distance metric. This groups the objects into clusters which are not separable linearly. Here radial basis kernel function is applied for sFCM clustering. The proposed clustering algorithm is tested on MRI image and noise induced MRI image. The results reveal that kernel based spatial fuzzy c means (skFCM) is better than Euclidean based spatial fuzzy c means

Received on: 30<sup>th</sup>-Nov-2015

Revised on: 11<sup>th</sup>-March-2016

Accepted on: 26- March-2016

Published on: 20<sup>th</sup>-May-2016

### KEY WORDS

Clustering, spatial, kernel, fuzzy sets, DB and D index, image segmentation

\*Corresponding author: Email: [deepthiph@gmail.com](mailto:deepthiph@gmail.com) Tel: +91-9986387435

## INTRODUCTION

Image processing is a rapidly growing field of which image segmentation forms a major part. It has diverse field of applications. Some of them are object recognition, machine vision and medical imaging. Development of segmentation algorithms which are efficient and insensitive to noise has become a challenge. Hence, it has become the need of the hour to develop better algorithms in the field of image segmentation. They must also be capable of solving real world applications and which are sensitive to noise. Noise in real world images are inevitable[1].

Conventional FCM algorithm fails to provide appropriate results on images which have noise. Spatial FCM (sFCM) [2], is a two step procedure. It includes the neighborhood information of pixel taken for study. Though it fails in completely removing the distortion of noise, the algorithm proves to handle noise more efficiently than conventional FCM[3].

The sFCM uses Euclidean distance formula for finding the spatial data point distances. It is found in literature that Euclidean distance fails to provide good results in situations where clustering algorithms are distance based. However, kernel methods provide better results as compared to Euclidean. This paper uses kernel distance formula and compares it with the results from Euclidean. This is hence an extension to the existing spatial FCM.

## DISTANCE METHODS

### EUCLIDEAN DISTANCE

Suppose  $a = (a_1, a_2, \dots, a_n)$  and  $b = (b_1, b_2, \dots, b_n)$  are two points in the n-dimensional Euclidean space. Then the Euclidean distance  $d(a, b)$  between a and b is given by

$$d(a,b) = \sqrt{\sum_{i=1}^n (a_i - b_i)^2} \quad (1)$$

### KERNEL DISTANCE

Let 'a' denote a data point. Then transformation of 'a' to the feature plane which possess higher dimensionality be denoted by  $\phi(a)$ . Description of inner product space is given by  $K(a,b) = \langle \phi(a), \phi(b) \rangle$ . Let  $a = (a_1, a_2, \dots, a_n)$  and  $b = (b_1, b_2, \dots, b_n)$  are two points in the n-dimensional space. Kernel functions used in this paper are stated as follows

- Radial Basis Kernel

$$R(a,b) = \exp\left(-\frac{\sum_{i=1}^n (a_i^p - b_i^p)^q}{2\sigma^2}\right) \quad (2)$$

Implementations of all the algorithms corresponding to Radial Basis Kernel have been done using p=2 and q=2 in equation (2).

- Gaussian Kernel (RBF with p=1 and q=2)

$$G(a,b) = \exp\left(-\frac{\sum_{i=1}^n (a_i - b_i)^2}{2\sigma^2}\right) \quad (3)$$

- Hyper-tangent Kernel

$$H(a,b) = 1 - \tanh\left(-\frac{\sum_{i=1}^n (a_i - b_i)^2}{2\sigma^2}\right) \quad (4)$$

where  $\sigma^2 = \frac{1}{N} \sum_{i=1}^N \|a_i - a'\|^2$  and  $a' = \frac{1}{N} \sum_{i=1}^N a_i$

For all kernels functions, N denotes total number of existing data points and  $\|x-y\|$  denotes Euclidean distance between points x and y which pertain to Euclidean metric space. By  $D(a, b)$  denotes the complete form of kernel distance function where  $D(a, b) = K(a, a) + K(b, b) - 2K(a, b)$  and when similarity property (i.e.  $K(a, a) = 1$ ) is applied, the following is obtained

$$D(a,b) = 2(1 - K(a,b)) \quad (5)$$

### EXISTING METHODS

Fuzzy models are incorporated in analysing spatial data.

#### Fuzzy Clustering

James C Bezdek developed fuzzy set based Fuzzy c-mean algorithm[4,5]. In this clustering method, each element can belong to more than one cluster. Each element is also associated with a set of membership values. Fuzzy clustering process involves assigning every data element to one or more than one cluster by taking into account their membership values[6,7].

1. Assign initial centers for c clusters.
2. Calculate distance  $d_{ik}$  between data objects  $x_k$  and centroids  $v_i$  using Euclidean formula

$$d(x, y) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + \dots + (x_n - y_n)^2}$$

3. Generate the fuzzy partition matrix or membership matrix U:

If  $d_{ij} > 0$  then

$$\mu_{ik} = \frac{1}{\sum_{j=1}^c \left(\frac{d_{ik}}{d_{jk}}\right)^{\frac{2}{m-1}}} \quad (6)$$

Else

$$\mu_{ik} = 1$$

4. The cluster centroids are calculated using the formula

$$V_i = \frac{\sum_{j=1}^N (\mu_{ij})^m x_j}{\sum_{j=1}^N (\mu_{ij})^m} \quad (7)$$

5. Calculate new partition matrix by using step 2 and 3
6. If  $\|U^{(r)} - U^{(r+1)}\| < \epsilon$  then stop else repeat from step 4.

Usually, for all experimental purpose, m is considered to be 2 and  $\epsilon$  to be 0.02.

## SPATIAL KERNEL FUZZY C MEANS (sKFCM)

Chuang, et al [2] developed spatial Fuzzy C means (sFCM) which incorporated spatial data of the image. It was a modification to the conventional FCM. The spatial function is given by the summation of the membership values in the neighboring of every pixel under consideration. The advantages were that they yielded more homogeneous clusters, diminished spurious blobs and boisterous spots and is insensitive to noise when compared to other systems. On comparable lines, spatial IFCM was additionally developed by Tripathy et al [8] by presenting the intuitionsitic feature.

Kernel functions add an added advantage to clustering [9,10]. In general, when two beside pixels are considered, correlation between them is relatively high. Since the neighboring pixels share similar intensity, the probability of them getting grouped into a same cluster is extremely high. The spatial FCM algorithm exploits this criteria. A spatial function is defined as:

$$h_{ij} = \sum_{k \in NB(x_j)} u_{ik} \quad (8)$$

where  $NB(x_j)$  gives neighborhood pixels of  $x_j$ . A mask of 5x5 which is equally weighted is used having pixel  $x_j$  as it's center. Spatial function  $h_{ij}$  portrays the likeliness degree of  $x_j$  is in  $i^{\text{th}}$  cluster. The spatial function values is usually high if most of the pixels in the neighborhood of a particular pixel belong to the same cluster. It is included in the membership function as:

$$u'_{ij} = \frac{u_{ij}^p h_{ij}^q}{\sum_{k=1}^c u_{kj}^p h_{kj}^q} \quad (9)$$

Here p and q denote the relative weightage of the initial membership and the spatial function respectively. The spatial kernel FCM with parameters p and q is denoted by sKFCM<sub>p,q</sub>. In noisy image conditions, the spatial function reduces the number of misclassified pixels by taking the neighboring pixels into account.

The sKFCM clustering algorithm has two-steps. For every iteration, the conventional FCM algorithm is followed in the first step. Here the distance formula being used is radial-based kernel distance. Later, the centroid and the membership functions are updated. In the second step, the spatial function  $h_{ij}$  is calculated and then the new membership function (6) is computed.

## RESULTS

A 225x225 dimension brain MRI image has been considered for implementation of sKFCM. We have considered the number of clusters,  $c=3$ . **Figure- 1** denotes the original image and **figure- 2** is the image induced with speckle noise with mean 0 and variance 0.04.



**Fig:1. Original MRI image      Fig:2. MRI image with speckle noise**

$V_{pc}$  and  $V_{pe}$  indicate fuzzy partition coefficient and partition entropy respectively. Maximum  $V_{pc}$  and minimum  $V_{pe}$  indicate better clustering [2]. DB and D indices are used to measure the cluster quality [11,12]. Higher  $V_{pc}$  and lower  $V_{pe}$  indicate a good clustering. FCM, sFCM and sKFCM have been applied to both the images.  $V_{pc}$  and  $V_{pe}$  is given by

$$V_{pc} = \frac{\sum_j^N \sum_i^c u_{ij}^2}{N} \quad (10)$$

and

$$V_{pe} = - \frac{\sum_j^N \sum_i^c [u_{ij} \log u_{ij}]}{N} \quad (11)$$

### DAVIS-BOULDIN (DB) INDEX

The DB index is defined as the ratio of sum of within-cluster distance to between-cluster distance. It is formulated as given.

$$DB = \frac{1}{c} \sum_{i=1}^c \max_{k \neq i} \left\{ \frac{S(v_i) + S(v_k)}{d(v_i, v_k)} \right\} \quad \text{for } 1 < i, k < c \quad (12)$$

This index tries to minimize the within cluster distance and maximize the between cluster separation. Therefore a good clustering procedure should give value of DB index as minimum as possible [11].

### DUNN(D) INDEX

Similar to the DB index the DD index is used for the identification of clusters that are compact and separated. It is computed by using

$$Dunn = \min_i \left\{ \min_{k \neq i} \left\{ \frac{d(v_i, v_k)}{\max_l S(v_l)} \right\} \right\} \quad \text{for } 1 < k, i, l < c \quad (13)$$

This tries in maximizing the between-cluster distance and minimizing the within-cluster distance. Hence a larger value for the D index proves clustering to be more efficient [13].

The results of the validity measures on original image are shown in **Table- 1**.

Table:1.Cluster Evaluation Results On Normal Image

METHOD	RESULTS ON ORIGINAL IMAGE			
	$V_{pc}$	$V_{pe}$	DB	D
FCM	0.7107	$1.5350 \times 10^{-4}$	0.2581	5.0521
sFCM <sub>1,1</sub>	0.7151	$3.4602 \times 10^{-09}$	0.2553	5.3562
sFCM <sub>2,1</sub>	0.7191	$1.7579 \times 10^{-13}$	0.2603	5.1039
sFCM <sub>1,2</sub>	0.7159	$6.4532 \times 10^{-14}$	0.2592	5.399
sKFCM <sub>1,1</sub>	0.727	$5.2001 \times 10^{-32}$	0.2492	5.6926
sKFCM <sub>2,1</sub>	0.7013	$4.3964 \times 10^{-43}$	0.2537	5.3698
SKFCM <sub>1,2</sub>	0.7276	$4.7799 \times 10^{-48}$	0.2501	5.7399

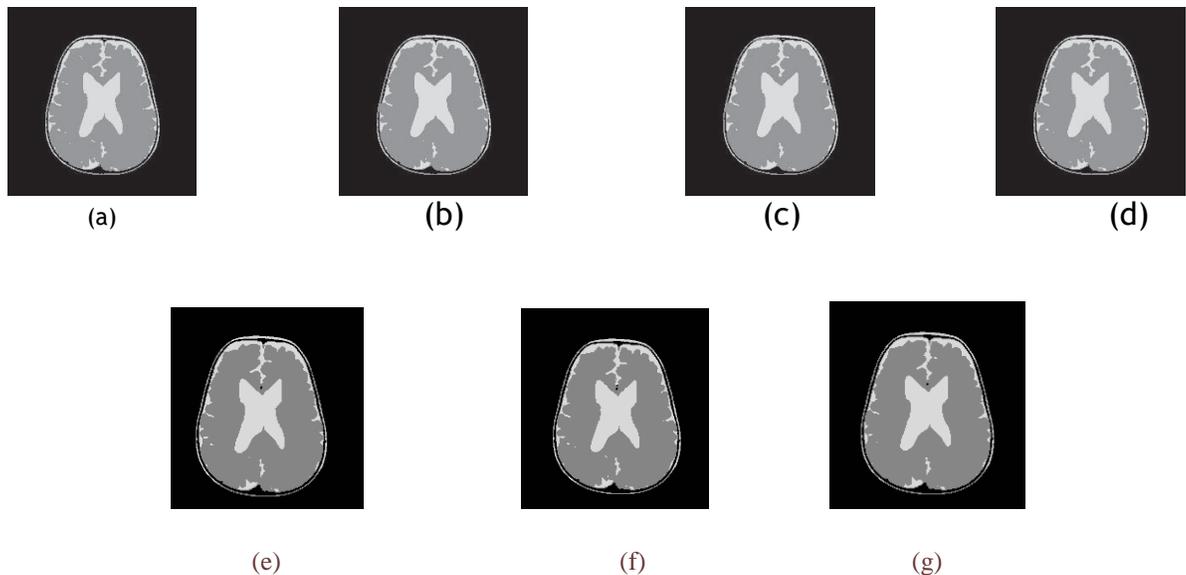


Fig: 3. Image segmentation of original image. (a) FCM. (b) sFCM<sub>1,1</sub>.(c)sFCM<sub>1,2</sub>.(d)sFCM<sub>2,1</sub>.(e)sKFCM<sub>1,1</sub>.(f)sKFCM<sub>1,2</sub>.(g)sKFCM<sub>2,1</sub>

From the above table and images, it can be seen that sKFCM has better partition coefficient and also possesses less partition entropy. sKFCM also has lower DB and higher D value when compared to conventional FCM and sFCM, thereby, sKFCM provides better clustering.

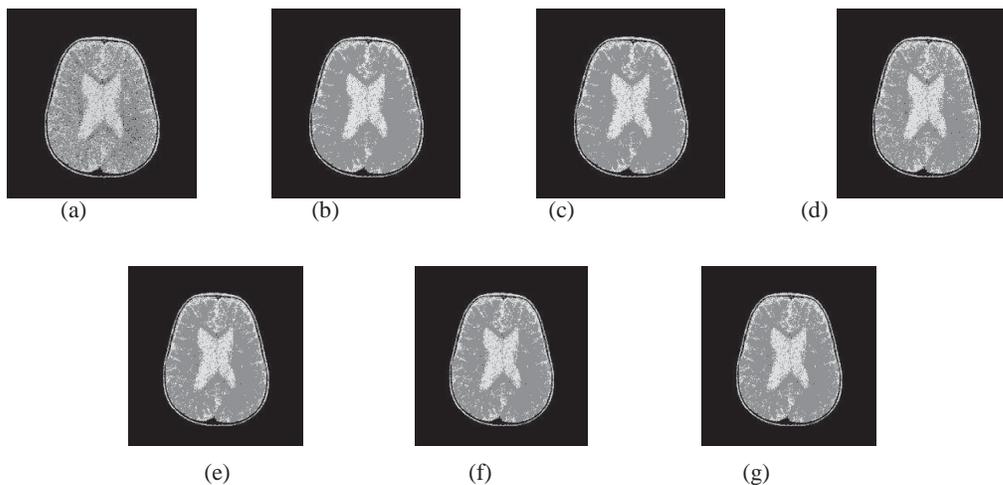
In the scenario of image with noise, the results are proved to be much better. Conventional FCM does not cluster the image to the expected level in presence of noise. Hence, leads to misclassification. The table below shows the performance of the sKFCM with other techniques implemented on the noisy image.

Table: 2. Cluster Evaluation Results On Noisy Image

METHOD	RESULTS ON NOISY IMAGE			
	$V_{pc}$	$V_{pe}$	DB	D
FCM	0.6975	$2.8195 \times 10^{-4}$	0.4517	3.4183
sFCM <sub>1,1</sub>	0.7101	$5.9541 \times 10^{-9}$	0.4239	3.6734
sFCM <sub>2,1</sub>	0.6922	$7.7585 \times 10^{-12}$	0.4326	3.4607

<b>sFCM<sub>1,2</sub></b>	0.6874	$4.2711 \times 10^{-12}$	0.4412	3.6144
<b>sKFCM<sub>1,1</sub></b>	0.7432	$1.3488 \times 10^{-25}$	0.3708	4.0343
<b>sKFCM<sub>2,1</sub></b>	0.7309	$3.7309 \times 10^{-31}$	0.3839	4.1718
<b>sKFCM<sub>1,2</sub></b>	0.7086	$7.3218 \times 10^{-37}$	0.3832	3.9454

From the **Table- 2** and **Figure- 4**, it can be seen that sKFCM produces better results. For noisy image, sKFCM overpowers FCM and all other forms of sFCM. sKFCM reduces the number of spurious spots and blobs to a large extent. It produces a segmented image with a good homogeneity. Smoother segmentation is achieved by taking a high value of  $q$ . But disadvantage is that, it may blur some of the finer details. The below figures show the segmentation results of sKFCM on the image induced with speckle noise.



**Fig: 4.**Image segmentation of noisy image (a)FCM(b)sFCM<sub>1,1</sub>(c)sFCM<sub>1,2</sub>(d)sFCM<sub>2,1</sub>(e)sKFCM<sub>1,1</sub>(f)sKFCM<sub>1,2</sub>(g)sKFCM<sub>2,1</sub>

## CONCLUSION

The proposed method adds a kernel approach to the conventional sFCM algorithm. It can be seen that the results of image segmentation and clustering by this approach has brought in better results. The Euclidean distance fails when clustering is to be done where distance is the major parameter. It is when kernel distance provides better results. Proposed method provides a novel way of clustering. The other kernel techniques like Gaussian and Hyper Tangent were also applied during the study. But only radial basis kernel's results were significantly different. However, the reason for unsatisfactory results of Gaussian and hyper tangent kernels is an area open for study. Likewise, this area calls research for developing hybrid clustering algorithms based on uncertainty.

## FINANCIAL DISCLOSURE

No financial support was received to carry out this research.

## ACKNOWLEDGEMENT

None.

## CONFLICT OF INTERESTS

Authors declare no conflict of interest.

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## ABOUT AUTHORS



*Deepthi P Hudedagaddi is pursuing her Masters at Vellore Institute of Technology. She is working on fuzzy clustering techniques on spatial data.*



*Dr. B K Tripathy, a triple gold medalist, is a senior professor in VIT University. He has supervised 19 PhD s, 13 M. Phil s and 02 M.S degrees. He is a senior member of IEEE, ACM, ACEEE and CSI. and is associated with over 60 international journals,published 320 articles, two research volumes and two books.He is working on Rough sets, Fuzzy sets, Social networks, Data mining, Soft Computing, E-Learning, Granular computing, Multi criteria decision making, Neighbourhood systems, SloT and Soft Sets.*