

ARTICLE

THE USE OF DYNAMIC GEOMETRY SYSTEMS AS A MEANS OF VISUAL THINKING ACTIVATION FOR STUDENTS WHO STUDY MATHEMATICAL ANALYSIS

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ABSTRACT

At present, software products are increasingly being used in universities for teaching various disciplines for both professional and educational purposes. The Federal State Educational Standard of Higher Education in the Russian Federation presents new requirements related to the application of modern information technologies to learning outcomes. In this regard, Russian higher education enters a qualitatively new level: the task of widespread use of information technologies in general and vocational education is being solved, which contributes to the modernization of the educational process, activating the thinking activity of the students, contributing to the development of the creativity of teachers, thereby increasing the effectiveness of the educational process. The goal of informatization is the global intensification of intellectual activity through the use of new information technologies, such as: computer, multimedia and interactive. This is especially relevant in the teaching of mathematical disciplines. The paper proposes a technique related to the systematic use of mathematical packages in the process of teaching mathematical analysis. This technique allows the use of dynamic geometry software systems (Dynamic Geometry System) to overcome the difficulties associated with spatial representation, which arise when studying applications of definite integrals. The suggested teaching methodology based on the use of Dynamic Geometry System Geo Gebra and multimedia technologies includes: presentation of theoretical material, step-by-step construction of the required object, calculation of body volume as consisting of parallel sections. The article presents solutions on the topic "Defined Integral" using the Geo gebra package implementing the proposed technique.

INTRODUCTION

The federal state educational standard of higher education, approved by the order of the Ministry of Education and Science of Russia, imparts to the learning outcomes new requirements related to the use of modern computer systems [1]. In connection with this, at present Russian higher education gets a qualitatively new level: the task of mass use of computer and information technologies in general and professional education is being solved.

The teaching of mathematics under modern conditions has begun to undergo major changes towards in terms of decreasing the number of hours given, since the development of science makes it necessary to introduce new disciplines into the learning process. This leads to a reduction and a more superficial presentation of the material of the basic mathematical courses. At the same time, educational facilities in mathematics practically do not change. The problem arises: how to make mathematical education more effective. To solve it, it is necessary to pay serious attention to the methodology of teaching mathematical disciplines using information technologies [2].

"In the field of teaching mathematics, there is the problem of integrating fundamental mathematical education and the very methodology of teaching mathematics, taking into account the current trends in the growth of the influence of information technologies" [3].

The solution of this problem is realized in the papers, where issues of mathematical - computer modeling and also geometric modelling are covered [4,5].

The use of multimedia technologies and systems of computer mathematics (SCM) makes it possible to include a wide visual series in the educational process, thereby activating the figurative thinking of students and helping them to perceive the proposed material in a holistic manner. The teacher has the opportunity to combine the presentation of theoretical information with showing the visual material. The use of multimedia technologies allows the teacher to manage the demonstration of visual material much more effectively, "opening new opportunities in the organization of the learning process, and also in developing the creative abilities of the students" [6].

Particularly topical was the use in the higher school of specialized software packages in the study of various disciplines [7,8,9,10], which are based on the principle of visibility. In accordance with this principle, specific geometric constructions are used in teaching, visualization of which makes it possible to easily solve the problems posed. The ability to visually represent geometric objects is a key feature of systems of dynamic geometry.

Many qualitative studies have been carried out both in software [11] of dynamic geometry systems and in the effective use of these systems in educational process.

For example, it has been shown in [12,13,14] that one of the effective ways to solve a number of problems in modern mathematical education is the use of the Geo Gebra dynamic geometry system.

KEY WORDS

Definite integral,
mathematical software,
principle of visualization

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MATERIALS AND METHODS

Statement of the problem

In this paper, we propose a methodology for teaching a section of mathematical analysis related to applications of certain integrals in calculating body volumes using information technologies.

The perception of this material is simplified with a reasonable use of visual opportunities provided by information technologies. The researched objects are demonstrated using multimedia technologies and computer mathematics systems, which allows the teacher to combine the material with a dynamic video sequence. This is a simple and logical explanation - the means that the computer provides for displaying information, surpass any printed matters. In no textbook, you cannot represent the object in dynamics. With such opportunities, the question arises related with the use of video materials.

Methodology

The complexity of the spatial representation and the absence of illustrative elements makes it impossible to see the objects researched entirely and makes it difficult to study applications of certain integrals. It is known that Dynamic Geometry (DGS) software systems can be useful tools in the study of this section, for example, DGSGeoGebra [15, 16] presents the possibility of step-by-step view of the solution of the problem and visualization of the theorems proofs.

In connection with the above, the teaching methodology based on the use of DGS Geo Gebra and multimedia technologies may include:

- presentation of theoretical material;
- step-by-step construction of the desired body;
- Calculation of body volume over areas of parallel sections.

The presentation of theoretical material [17] can be based on the use of presentations, video clips, dynamic video sequences, etc [Fig. 1].

| Calculation of body volume over areas of parallel sections | The limit of this sum under $\max \Delta x_i \rightarrow 0$ is called the body volume |
|--|--|
| <p>Find the volume V of the body T if we know the areas S of any section of the given body by planes perpendicular to the axis OZ. The area depends on the position of the cutting plane, i.e. is a function of x: $S=S(x)$.</p> <p>! $S(x)$ – continuous function</p> <p>Draw the planes</p> <p>! $x = x_0 = a, x = x_1, \dots, x = x_{i-1}, x = x_i, \dots, x = x_n = b$</p> <p>Let us construct a cylindrical body. Cylinder capacity:</p> <p>! $V_n = \sum_{i=1}^n S(\varepsilon_i)$</p> | <p>$V = \lim_{\max \Delta x_i \rightarrow 0} \sum_{i=1}^n S(\varepsilon_i) \Delta x_i$</p> <p>$V_n$ is an integral sum for a continuum function $S(x)$ on the stretch $[a, b]$, and the limit exists and is expressed by the definite integral:</p> <p>$V = \int_a^b S(x) dx$</p>  |

Fig. 1: The example of presentation of theoretical material.

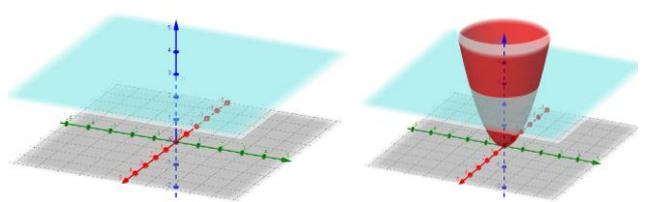
Step-by-step construction of the desired body involves the use of computer mathematics systems that allow a dynamic review of the problem solution. Consider the application of this methodology on the basis of the example of solving the following problems [3].

Example 1. Find the volume of the body bounded by the paraboloids surface $x^2+y^2=z$ and the plane $z=3$.

In [Table 1], in points 1–3, the steps for constructing the target body in DGS Geo Gebra are presented.

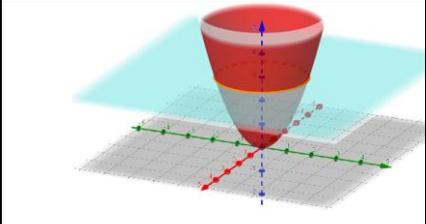
Table 1: The example of step-by-step construction

1. We construct the plane $z = 3$ and the paraboloid $x^2 + y^2 = z$



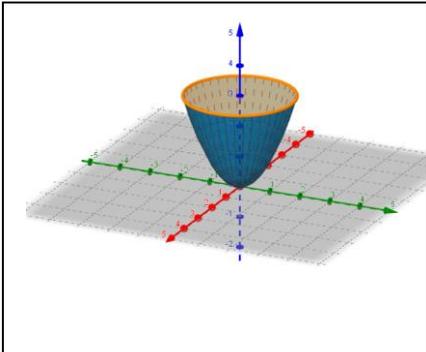
- Paraboloid
 - a: $x^2 + y^2 + 0z^2 - z = 0$
- Plane
 - b: $z = 3$

2. Find their intersection using the tool



- Conic
 - c: $X = (0, 0, 3) + (1.73 \cos(t), 1.73 \sin(t), 0)$

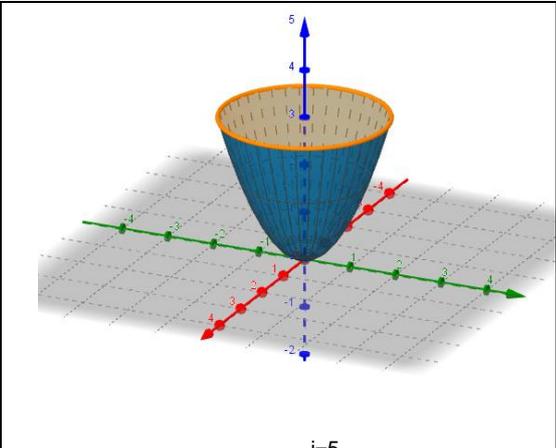
3. We describe the body, the volume of which must be found.



- Surface
 - $e(c, t) = \begin{pmatrix} t \cos(c) \\ t \sin(c) \\ t^2 \end{pmatrix}$
- Parametric curve
 - d: $\left. \begin{matrix} y = \sqrt{3} \sin(t) \\ z = 3 \end{matrix} \right\} -3.14 \leq t \leq \pi$

We note that for step-by-step viewing of the solution of the problem and dynamic visualization of the construction of the desired body, it is necessary to create a slider and determine the conditions for displaying body components for objects with different values of the slider. For example, create the slider i with an interval from 0 to 6 (step 1) [Table 2].

Table 2: Settings of slider



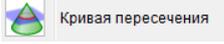
In the "Object Properties" slider dialog box, set the animation behavior: speed -1; repeat - increase (once). The objects of the desired body (point 3) are displayed at $4 \leq i \leq 6$, and the previous constructions (points 1-2) correspond to the values $0 \leq i \leq 3$.

Using the results of the construction, we find the volume by means of the integral:

$$V = 4 \cdot \int_0^{\sqrt{3}} dx \int_0^{\sqrt{3-x^2}} (3-x^2-y^2) dy = \frac{9\pi}{2}$$

Example 2. Find the volume of the body bounded by the cylindrical surface $x^2 + y^2 = 5$ and the planes: $z=0$ and $y+z=3$.

When calculating body volume over areas of parallel sections, we use dynamic visualization, which is realized by introducing the slider t with an interval from $-\sqrt{5}$ to $\sqrt{5}$ in steps of 0.1.

To move the section along the Ox axis, you need to join the x coordinate with the value of the slider t . Set the section using the tool  .

In this example, the section is a polygon, so it is convenient to construct it by specifying the coordinates of the vertices parametrically:

$$\begin{aligned} A(t, \sqrt{5-t^2}, 0); & B(t, -\sqrt{5-t^2}, 0); \\ C(t, -\sqrt{5-t^2}, 3 + \sqrt{5-t^2}); & \\ D(t, \sqrt{5-t^2}, 3 - \sqrt{5-t^2}) & \end{aligned}$$

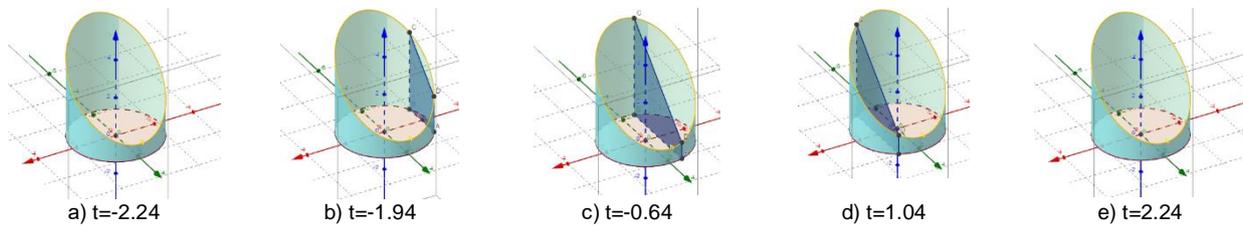


Fig. 2(a-e): The dynamic visualization of sections.

Using the results of the construction, we find the volume by means of the integral.

$$V = 4 \int dx \int (3-y) dy = 15\pi$$

RESULTS AND DISCUSSION

Perception of the teaching material depends on the way how the educational process is organized, what is aimed at. That is why it is necessary to pay serious attention to the teaching methods of mathematical disciplines using information technology.

The presented method is based on the systematic usage of information technology during teaching process of higher mathematics. Mathematical packages, including a dynamic geometric environment - the Dynamic Geometry (DGS) software systems are successful approaches for a teacher. They combine both multimedia and programming tools, thereby providing informatization of the educational process.

The following results were obtained during the research:

- using of multimedia technologies assists to improve the level of understanding of complex sections of higher mathematics;
- dynamic Geometry Systems allows to make geometric constructions of the explored objects visualization of which help to solve tasks and view them in dynamics;
- using of multimedia technologies and systems of computer mathematics helps to optimize the educational process, using time at different stages of teaching more rationally;
- a method based on the systematic using of IT contributes to a deep understanding of the subject and the development of interest in its study.

Using of information and communication technologies has a great influence on the content, forms and methods of teaching. New educational technologies which are used during the learning process improves the teaching methods, allows using the visual possibilities provided by computer mathematics systems along with traditional methods, techniques and methods.

The article proposes a technique for teaching a section of mathematical analysis related to applications of definite integrals during calculating of volume of shapes using multimedia technologies and graphical capabilities of dynamic geometry systems. The usage of the proposed method of teaching showed that

DGS Geo Gebra is a necessary tool to build geometric constructions of the explored objects, the visualization of which help to solve tasks for calculating volume of shapes.

CONCLUSION

The introduction of this technique is recommended due to the current level of development of information technology, as well as problems associated with the using of visualization of spatial modeling of objects. Using of multimedia technologies allows a teacher to diversify the study material with new activities, enrich it with visual information, increase the motivation of students and their interest to the subject. Students actively participate in the learning process, are trained to think independently, to protect their own points of view, to model real mathematical objects.

The creation and development of methods based on the systematic using of information technology is a necessary step towards increasing the effectiveness of mathematical education.

CONFLICT OF INTEREST

There is no conflict of interest.

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None

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