KOCH FRACTAL DIPOLE MATCHING BY TURNING OF THE ARMS

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ABSTRACT

A dipole wire antenna of the Koch is considered. The antenna represents a wire dipole symmetrical with respect to the point of feeding. Arms of the dipole have the geometry of Koch prefractal. A family of antennas is singled out, in which the antennas differ from each other by an angle of arms turning. Antennas having the geometry of the first two iterations of a Koch curve are chosen for the analysis. Dipoles based on the Koch pre-fractals of the first two iterations having different wire thicknesses obtained by rotating one of the arms around a given coordinate axis is considered. Graphs depicting dependence of the reflection coefficient at the base frequency on angles of rotation around the axes are presented. Resonance angles of rotation of a “perfectly” matched antenna’s arms are obtained. A conclusion is drawn on possibility of significant improvement of antenna matching by turning the dipole arms with respect to each other.

INTRODUCTION

A classical symmetric electric dipole containing two identical arms fed in the middle represents one of the most well explored objects in the theory of antennas [1]. There exist various methods of improvement of electrodynamic characteristics of such dipoles [2]. For example, the improvements can come from varying an angle between the arms and obtaining the so-called V-shaped antennas [3] as well as from mutual influence of radiation originating from the arms on each other. Another method of improvement of the symmetric dipole's characteristics is related to placing the arms in an antisymmetric manner. The method was applied in [4] for improving several resonance characteristics of the classical Koch and quadratic Koch fractal dipole antenna; the study also compared the obtained antennas with each other. A dual-band dipole antenna with asymmetric arms was presented in [5] for WLAN applications.

For improving the electrodynamic characteristics of dipoles, modification of the ratio of sizes (areas) of the dipole’s arms is also utilized. For example, study [6] considered a dipole antenna consisting of two printed strips of unequal lengths and presented graphs for reflection coefficients and radiation patterns of the proposed antenna. A balanced-to-unbalanced transformer or balun is also often used [7]. The study [8] considered a printed dipole antenna with a micro strip balun and demonstrated an influence of sizes of the balun on return loss.

On can also make use of multiple folded arms. For example, the folded dipole of two and more elements was used as an impedance transformer to match the antenna to a line of higher characteristic impedance in [9]. In addition, multi armed related structures are also used. The studies [10, 11] considered antennas consisting of an axially symmetric array of four and six conductor arms forming a spherically shaped structure and demonstrated influence of increase in the number of arms on quality factor and bandwidth.

For reducing the sizes of the dipoles, one can alter topology of the arms, so that the electric length of the antenna increases, while radius of the sphere covering the dipole remains unchanged. For that purpose, one can roll the arms into a helix [12, 13] or perform various fractal transformations [14]. In this regard, the following antennas can be specified: an antenna which is based on the Koch curve [15, 16], Minkowski curve [17, 18], Sierpinski carpet [19, 20], a rounded fractal antenna [21, 22] and complex fractal combinations [23-26].

Characteristics of the dipole can be further improved by rotating its arms in space with respect to each other. For example, properties of the Koch dipole during rotating the arms in one plane (in the antenna's plane) were studied in [27]. In the present study, we analyze the effect of rotation of the Koch dipole's arms around its axes on the reflection coefficient at the base frequency on angles of rotation around the axes. As a result, we present graphs and draw the relevant conclusions. We draw a conclusion that that by turning the arms by a certain angle one can essentially improve the antenna matching.

STATEMENT OF THE PROBLEM

The first fractal antenna, whose electromagnetic and directional properties were studied most completely and extensively, was the antenna based on the pre-fractal Koch curve. When constructing a Koch line, the initial interval of length L_0, referred to as an initiator of the fractal, is split into three equal parts. The central part is replaced with an equilateral triangle having sides of length L_0/3. As a result, there...
appears a broken line consisting of four links; each of the links has length $L_0/3$ [Fig. 1]. The process is carried out for each segment of the broken line.

We consider symmetric dipoles based on the Koch pre-fractal (DBKP) of the first two orders. The zeroth order DBKP coinciding with the ordinary dipole is shown in [Fig. 1(a)], the first order DBKP is shown in [Fig. 1(b)] and the second order DBKP is shown in [Fig. 1(c)]. The feed point for all the dipoles is located exactly in the middle.

Fig. 1: Symmetric Koch fractal dipole. Arm length $l=7.5$ cm, wire radius $r=1$ mm. a) zeroth-order dipole (ordinary dipole); b) first-order dipole; c) second-order dipole.

We assume that initially the antenna lies in the plane $z=0$. We rotate the right arm in space relative to the starting position [Fig. 2]. Moreover, we consider three sub-problems, each of which represents rotation around a particular axis [Fig. 2(a)-(c)].

We seek the reflection coefficient of the obtained new dipoles by rotating an arm. It should be noted that rotation around the $x$-axis does not influence electromagnetic characteristics in vicinity of the main resonance. A difference shows up only at higher frequencies in vicinity of the second resonance. The same conclusion regarding comparison of the symmetric and antisymmetric dipoles was drawn in [4]. Therefore, we will hereafter explore only rotations around the $y$-axis and $z$-axis.

Fig. 2: Options for rotation of an arm around the axes: a) rotation around the $x$-axis (twisting); b) rotation around the $y$-axis; c) rotation around the $z$-axis.

Calculations in the present work were carried out using the FEKO software. In all cases, length of the segment partitions was selected to be five times greater than radius. It was assumed that the wire was made of copper of a circular cross-section. Main calculations were conducted for wires having radiuses $r$ equal to $r=1.0$ mm, $r=1.5$ mm and $r=2.0$ mm.
For numerical implementation, we perform cycles with angle steps equal to 1-2 degrees. For determining the resonance angles with high accuracy, we decrease the angle step size to 0.01 degrees and frequency step size to 10 kHz. It should be noted that such a high accuracy theoretically allows determining the optimum rotation angle guaranteeing a perfect match. However, for all practical purposes, large steps are of more interest due to the fact that it is not so easy technically to perform rotations with accuracy of one hundredths of a degree.

**REFLECTION COEFFICIENT**

Let us consider a dependence of values of the first (main) minimum of the reflection coefficient $S_{11}$ on location of the arms. When changing the radius $r$ from 1.0 mm to 1.5 mm for the ordinary dipole, the minimum is reached at the value of angle $\alpha$ equal to approximately $78^\circ$. The angle $\alpha$, corresponding to the minimum of the reflection coefficient $S_{11}$, slightly increases with increase in $r$ and reaches the value $\alpha \approx 80^\circ$ at $r=2.0$ mm. The value of $S_{11}$ itself remains at the value close to -45 dB.

![Fig. 3: Dependence of minimum of $S_{11}$ on $\alpha$. The arm's length $l$ is 7.5 cm. The solid line corresponds to radius of the wire $r=1.0$ mm and the DBKP of the first order; diamond signs correspond to $r=1.5$ mm and the DBKP of the first order; dotted line corresponds to $r=2.0$ mm and the DBKP of the first order; circle signs correspond to $r=1.0$ mm and the DBKP of the second order; square signs correspond to $r=1.5$ mm and the DBKP of the second order; triangle signs correspond to $r=2.0$ mm and the DBKP of the second order. The dashed line corresponds to the ordinary dipole having radius $r=1.0$ mm. $l=7.5$ cm.](image)

A similar pattern is also observed for the first order DBKP when rotating around the y-axis. For smaller values of $r$, extreme values of $S_{11}$ are achieved at smaller values of the angle $\alpha$. The values of $\alpha$...
continuously increase from $26^\circ$ to $38^\circ$ when changing $r$ from 1.0 mm to 2.0 mm. The minimum values remain at the level $S_{11} \approx -55$ dB.

Table 1: Extreme values of $S_{11}$ when rotating around the y-axis

<table>
<thead>
<tr>
<th>Radius $r$, mm</th>
<th>Angle $\alpha$, $^\circ$</th>
<th>Minimum $S_{11}$, dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>25.07</td>
<td>-100</td>
</tr>
<tr>
<td>1.5</td>
<td>32.73</td>
<td>-96.5</td>
</tr>
<tr>
<td>2.0</td>
<td>38.14</td>
<td>-95.5</td>
</tr>
</tbody>
</table>

At larger values of the angle $\alpha$, the difference between antennas, having different radiuses, practically disappears. The reflection coefficient of the first-order DBKP for $\alpha \to 90^\circ$ fluctuates around the value -13 dB.

For the second order DBKP [Fig. 3, 4], the reflection coefficients take on larger values at smaller radiuses. This means that the dipoles of the second order are better matched for large values of $r$. As the angle $\alpha$ increases [Fig. 3], the minimum value of $S_{11}$ monotonically increases for the second order DBKP. The exact values obtained for a finer grid (angle step size is 0.01$^\circ$ and frequency step size is 20 kHz) are presented in [Table 1]. Note that for an ordinary dipole, the exact value of the minimum of the reflection coefficient $S_{11} \approx -90$ dB having radius of the wire 1 mm is achieved at the rotation angle of $77.68^\circ$.

Table 2: Extreme values of $S_{11}$ when rotating around the z-axis

<table>
<thead>
<tr>
<th>Radius $r$, mm</th>
<th>Angle $\beta$, $^\circ$</th>
<th>Minimum $S_{11}$, dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>-14.34</td>
<td>-120</td>
</tr>
<tr>
<td>1.0</td>
<td>+43.31</td>
<td>-99</td>
</tr>
<tr>
<td>1.5</td>
<td>-21.30</td>
<td>-99</td>
</tr>
<tr>
<td>1.5</td>
<td>+49.72</td>
<td>-97.5</td>
</tr>
<tr>
<td>2.0</td>
<td>-26.46</td>
<td>-98</td>
</tr>
<tr>
<td>2.0</td>
<td>+54.38</td>
<td>-97</td>
</tr>
</tbody>
</table>

Regarding the rotation around the z-axis, we can state only the following. First, we can draw the coordinate axis min $S_{11}$ not for the value $\beta=0^\circ$, but, instead, for the value $\beta=20^\circ$. Note that in this case, speculations concerning the rotations by angles greater than $20^\circ$ and by angles less than $20^\circ$ remain the same as speculations for the case of rotation around the y-axis.

Note that for the second order DBKP having the wire radius of 2 mm, the minimum of $S_{11}$ in the range $0^\circ < \beta < 30^\circ$ follows a complicated oscillatory pattern (triangles in [Fig. 4]). However, on the outside of the range, the curve of min $S_{11}$ for a given antenna demonstrates behavior, which is similar to rotation around the y-axis.

Extreme values of $S_{11}$ during rotation around the z-axis, determined with high accuracy, are presented in [Table 2].

BEST ANTENNAS

Let us consider radiation characteristics of the optimal dipoles. We present graphs depicting the reflection coefficients values for optimal antennas having the wire of radius $r=1$ mm. In line with [Tables 1 and 2], they correspond to antennas obtained at the rotation angles $\beta=-14.34^\circ$, $\beta=43.31^\circ$, and $\alpha=25.07^\circ$. It can be seen that behavior of the reflection coefficient at the first resonance frequency (around 810 MHz)
is practically identical for the three proposed antennas. Differences in graphs of S_11 show up at the second frequency around 2.4 GHz.

Analysis of electrodynamic characteristics, conducted at turning the antenna around various axes, allows stating the following. Turning the antenna around the x-axis does not alter electromagnetic characteristics of the first resonance frequency; influence of the turns becomes discernible only at higher frequencies (see, for example, [4]). Optimum angles of turning the antenna in the antenna's plane (around the z-axis) and optimum angles of turning the antenna around the y-axis produce almost identical very well matched antennas. Based on this observation, we can recommend turning the arms in the antenna's plane (around the z-axis) for the purpose of improving characteristics of the antenna. In this case, sizes of the antenna do not change, and matching of the dipole can be significantly improved.

Fig. 5: Graphs of S_11 for optimal antennas (l=7.5 cm; r=1.0 mm). The case of the DBKP of the first order. The solid line corresponds to $\beta=14.34^\circ$, the dashed line corresponds to $\beta=43.31^\circ$ and the dotted line corresponds to $\alpha=25.07^\circ$.

In addition, it should be noted that for some applications, it is required that the antenna must be stretched. From this viewpoint, a change in its geometry by means of turning in the plane of the antenna is undesirable. However, from the classical point of view, the surface area, occupied by the antenna, is expressed in terms of radius of a circle covering the antenna itself. In this case, turning the arms in the antenna's plane does not affect the area of the dipole.

Fig. 6: Dependence of the gain on the rotation angle $\beta$ for the DBKP of the first order. The solid line corresponds to r=1.0 mm; the dashed line corresponds to r=1.5 mm; the dotted line corresponds to r=2.0 mm. l=7.5 cm.

Dependence of the gain on the rotation angle $\beta$ for the DBKP of the first order. The solid line corresponds to r=1.0 mm; the dashed line corresponds to r=1.5 mm; the dotted line corresponds to r=2.0 mm. l=7.5 cm. From the physical point of view, the effect of improving the antenna matching is explained by mutual influence on each other of separate component segments of the antenna (more precisely, influence on each other of electromagnetic fields emitted by the segments). At certain angles, the influence turns out
to be optimal. A more widespread variant consists in changing geometry of the arms. In both cases, the physical effect turns out to be the same.

Let us present dependence of the gain on the main resonance frequencies for dipoles with the arms representing the Koch pre-fractal of the first order [Fig. 6], and for dipoles with the arms representing the Koch pre-fractal of the second order [Fig. 7]. The values of the gain change in the range from 1.48 to 1.6, reaching the maximum values for the dipole having arms, which remain in the initial position.

![Fig. 7: Dependence of the gain on the rotation angle β for the DBKP of the second order. The solid line corresponds to r=1.0 mm; the dashed line corresponds to r=1.5 mm; the dotted line corresponds to r=2.0 mm. l=7.5 cm.](image)

Note that dipoles of large diameters possess large values of the gains. The difference for the Koch pre-fractal of the second order is found to be stronger than that for the Koch pre-fractal of the second order.

One can expect that for dipoles having a more complex geometry, despite a more complicated behavior of influence of the dipole's elements on each other, there also exist "optimum" turnings of the arms. At the same time, the graphs depicting dependence of reflection coefficients on angles of rotations must become much more complicated in case of complications of geometry of the arms.

**SUMMARY**

Rotations of the arms around their own axis (the x-axis) do not exert any influence on electromagnetic characteristics in vicinity of the main resonance. The main effects occur in rotation around the axis of the feed line (y-axis) as well as in the antenna's plane (z-axis). In the case of the optimal rotation around the y-axis and the z-axis, "perfect" antenna matching can be achieved.

**CONCLUSION**

The antenna matching can be significantly improved by small angle rotations of the dipole arms. It should be noted that rotation around the z-axis can be considered as more advantageous because it is carried out in the plane of the antenna, and rotations around the z-axis can be generalized to the case of the micro strip antennas.

**CONFLICT OF INTEREST**

There is no conflict of interest.

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None

**REFERENCES**


