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ABSOLUTE STABILITY ANALYSIS METHOD OF THE PROBING SIGNALS GENERATOR OF PHASE-METRIC SYSTEMS OF GEODYNAMIC CONTROL

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ABSTRACT

Phase-metric method of geo electric signals registration allows eliminating multiplicative noise and, accordingly, to increase the accuracy of geo electric control due to the use of phase methods of formation of probing signals. For the reliability of the phase-metric systems of geodynamic control it is necessary to ensure the stability of the signal generators of these systems at the geo-dynamic variations of the object of control and the effects of noise. When registering small geo-dynamic changes, the level of destabilizing effects sometimes exceeds the level of useful signal by 60 dB and more, which determines the need to use nonlinear model of the signal generator. A new method based on the V.M. Popov frequency criterion and the piecewise linear approximation of the hodograph is developed to study the absolute stability of the high-order generator model with an arbitrary value of perturbing effects. On the basis of the developed method, the stability analysis of probing signal formers with different types and orders of filters is performed. Studies had shown a significant difference between the calculated boundary regulation coefficients of the generator in linear and nonlinear regime.

INTRODUCTION

The phase-metric method of geo electric data registration is used to monitor the geo dynamics of near-surface in homogeneities in cases of the need to provide increased sensitivity to special changes in the object of study. High efficiency is achieved by increasing the sensitivity of the measuring system, initial installation and operational positioning of the electrical installation by controlling the sources of probing signals [1]. Technique of applying the phase-metric method of geo electric control is that in the case of the probing signal, several sources located in the field of the object under study and the required number of vector measurement sensors of the electric field are used. Point sources form probing signals shifted in phase by a given angle relative to each other. Geodynamic variations of the object under study are determined by the displacement of fictitious sources, which leads to the dis balance of the measuring system and the registration of the corresponding signal vector in it.

For reliability of the operation of phase-metric systems of geodynamic control is necessary to ensure the stability of the signal generators of these systems with variations in the installation parameters and the effects of noise. The article [2] develops a method and algorithm for analyzing the stability of the linear model of the former based on the Nyquist frequency criterion and the algebraic Raus-Hurwitz criterion, well known in the theory of automatic control [3, 4, 5]. The linear model of the device is adequate for small impact perturbations. At the same time, when recording small geodynamic changes, the level of destabilizing effects may be 60 dB or more higher than the level of useful signal [6, 7]. This determines the need to use a nonlinear model of the signal generator and, accordingly, other methods of stability analysis. Lyapunov methods [8, 9] enables to investigate the stability of devices operating in non-linear regime. The first method is based on the Lyapunov linearization of all nonlinear blocks for a particular equilibrium position. Therefore, its application is equivalent to the construction of a linear model for the maximum deviation of variables at the inputs of each nonlinear block [10]. Thus, the first Lyapunov method allows estimating only the asymptotic stability of the nonlinear former "in the small". The second Lyapunov method [8, 10] is universal, since it is not associated with linearization of the differential equation of a nonlinear device and does not impose any special restrictions on the nature of nonlinearity. However, application of the second Lyapunov method in practice is complicated by the lack of general recommendations on the choice of Lyapunov functions [10].

The frequency criterion of absolute stability of the equilibrium position was first proposed by V.M. Popov [11]. The implementation of this criterion provides asymptotic stability of the de-vice "as a whole", "in general" (at any deviations of the influencing disturbances). Popov's criterion is also applicable to devices whose nonlinear blocks have static characteristics with areas of ambiguity [12]. Another important advantage of the Popov criterion is the simple expression, clarity and convenience of the method not only for analysis, but also for the synthesis of nonlinear inertial devices. Approximation of the amplitude-phase characteristic (hodograph) allows simplifying the application of Popov frequency criterion for the study of the stability of devices with a complex form of inertial characteristics.

It is necessary to develop a new method for studying the stability of high-order signal formers (generators), which is applicable not only in the linear mode (for small influencing disturbances, "in small"), but also in the nonlinear mode (for an arbitrary value of the influencing disturbances, "in general"), within the

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framework of the computational experiment. The proposed method should combine the advantages of known algebraic and frequency criteria and the ability to study stability of the device not only for fixed values of the blocks parameters, but also when they change.

The purpose of this work is to develop a method for analyzing the absolute stability of the probing signal formers at any deviations of the influencing disturbances to ensure the reliability of the phase-metric method of geo electric control of geodynamic objects.

Phase formation of the probing signals of geodynamic control systems

Applied phase-metric method of geo electric signals registration allows to eliminate multiplicative noise and, accordingly, to increase the accuracy of geo electrical control through the use of phase methods for the formation of probing signals. Operation of phase-metric systems of geo electric control is based on the direct conversion of the useful signal into the phase of oscillation [7].

Probing signals of multipolar phase-metric systems can be represented with high accuracy either by single harmonic oscillations with defined amplitudes, frequencies and phases, or by a set of such oscillations. Thus, for the analysis of such installations, application of the model of amplitude-phase formation and transformation of signals is relevant [13, 14, 15]. The use of such a model is promising both for simplification of modeling and design of known monitoring systems and for modernization of known systems in order to increase their reliability and sensitivity by improving the quality of sounding signals formation and algorithms of their processing.

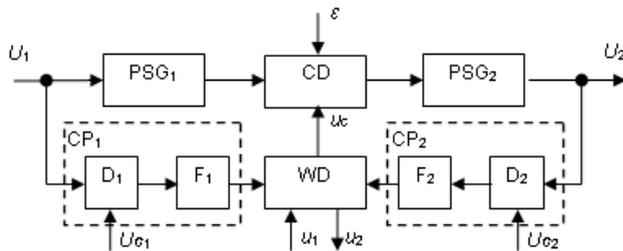


Fig. 1: Generalized PSG model.

The composition of the generalized model of the probing signal generator [PSG, Fig. 1], identical to the model [14], includes: similar to it PSG1,2, control device (CD), control paths (CP1,2) and a weight distributor (WD). Amplitude and (or) phase of the input signal of the generator is controlled in the control device (CD). Each control path consists of an amplitude and (or) phase deviation detector and a filter. Paths CP1 and CP2 implement the principle of perturbation and deviation, respectively. Values of transfer coefficients of the weight distributor determine the proportions of signal transmission from its inputs to outputs and allow to form the control and output auxiliary signals of the generator.

The scheme includes following designations: U1,2 - main input and output signals of the PSG, UG1,2 - signals of equivalent reference generators of detectors (D1,2), u1,2 - auxiliary input and output signals, Uc - control signal, ε - destabilizing factor.

Various options for the construction of generator - with perturbation control, control by deviation and combined control (CC) - we obtain by a simple choice of values of the corresponding coefficients of the weight distributor. Further expansion of PSG1,2 allows to represent devices by different number and type of connections (direct, inverse, local, general, multi-loop). Thus, flexible structure of the generalized model [Fig. 1] allows to investigate a wide class of schemes of signal generators of electro location systems of geodynamic control, distinguishing by the number of channels (poles) and dependence between the parameters of signals in individual channels, characteristics of components, magnitude and nature of the influencing disturbances, etc.

The method of absolute stability analysis of probing signals generators with arbitrarily large forcing perturbation

According to Popov's criterion [9, 11, 10], for absolute stability of the equilibrium position of a nonlinear system with a stable linear part, the existence of a real g is sufficient, for which the condition is satisfied

$$\forall \omega \geq 0: \text{Re}[(1 + j\omega g)W(j\omega)] > -1/k \quad (1)$$

where k is an angle of absolute stability, $W(j\omega) = \frac{A_1(\omega) + jA_2(\omega)}{B_1(\omega) + jB_2(\omega)}$ is the complex transfer function of the filter in the feedback circuit, and l is the order of the filter. The largest and the smallest values of k,

at which the condition (1) is satisfied, determine, respectively, the lower and up-per boundaries of the region of stable operation of the nonlinear generator.

We divide the complex transfer function of the filter $W(j\omega)$ into the real and imaginary parts:

$$W(j\omega) = W_R(\omega) + jW_I(\omega) \quad (2)$$

where $W_R(\omega) = \frac{A_1(\omega)B_1(\omega) + A_2(\omega)B_2(\omega)}{B_1^2(\omega) + B_2^2(\omega)}$ - real frequency characteristic of the filter F2,

$W_I(\omega) = \frac{A_2(\omega)B_1(\omega) - A_1(\omega)B_2(\omega)}{B_1^2(\omega) + B_2^2(\omega)}$ - imaginary frequency characteristic, polynomials

$A_{1,2}(\omega) = \sum_{i=0}^l A_{1,2,i}(\omega)$, $B_{1,2}(\omega) = \sum_{i=0}^l B_{1,2,i}(\omega)$ are determined according to the expressions:

$$A_{1,i}(\omega) = \text{Re}[A_i(j\omega)] = \alpha_{4i}\omega^{4i} - \beta_{4i+2}\omega^{4i+2},$$

$$B_{1,i}(\omega) = \text{Re}[B_i(j\omega)] = \beta_{4i}\omega^{4i} - \beta_{4i+2}\omega^{4i+2},$$

$$A_{2,i}(\omega) = \text{Im}[A_i(j\omega)] = \alpha_{4i+1}\omega^{4i+1} - \alpha_{4i+3}\omega^{4i+3},$$

$$B_{2,i}(\omega) = \text{Im}[B_i(j\omega)] = \beta_{4i+1}\omega^{4i+1} - \beta_{4i+3}\omega^{4i+3},$$

where α_i, β_i are the filter coefficients. Separation of the transfer coefficient of filter into the real and imaginary parts allows a simple geometric interpretation of the Popov criterion.

We introduce a modified complex transfer function

$$W^*(j\omega) = W_R(\omega) + jW_I^*(\omega) \quad (3)$$

where $W_I^*(\omega) = \omega W_I(\omega)$.

By converting (1) to (3), we obtain a sufficient condition of absolute stability in the form $W_R(\omega) - gW_I^*(\omega) > -1/k$. For the boundary values of the regulation coefficient, condition takes the form of equality (the Popov equation)

$$W_R(\omega) - gW_I^*(\omega) = -1/k \quad (4)$$

The straight line defined by equation (4) passes through the point $-1/k$ on the real axis with a slope of $1/g$.

To conduct an analytical study of the absolute stability of the PSG, it is necessary to obtain a Popov direct expression for a particular type of F2. Solution of the problem in a generalized form for an arbitrary type and order of filter is not possible. This difficulty arises due to the non-linear nature of the left side of equation (4) and the presence of two unknowns g and k . Approximation of the filter frequency characteristics based on continuous piecewise linear functions (CPLF) [14] allows one to linearize (4), to eliminate the unknown g and to conduct an analytical study of the absolute stability in general.

We set the following approximation parameters: the range of variation of the variable from ω_0 to ω_N , N - the maximum number of the approximation node, n, m - the current numbers of the approximation nodes. Frequency characteristics change most quickly in the region of ω small values and slowly - at large. Thus, in order to reduce error the exponential position of nodes is reasonable. Expressions of lines approximating the left side of (4) in the current nodes will take the form

$$W_{I\ m,n}^*(W_R) = g_{m,n}(W_R - b_{m,n}), \quad (5)$$

where $g_{m,n} = (W_{I\ m}^* - W_{I\ n}^*) / (W_{R\ m} - W_{R\ n})$ are angular coefficients, $W_{I\ m,n}^* = W_I^*(\omega_{m,n})$, $W_{R\ m,n} = W_R(\omega_{m,n})$,

$$b_{m,n} = W_{R\ m} - W_{I\ m,n}^* / g_{m,n} \quad (6)$$

- abscissas of the approximating straight lines.

The result is N^2 coefficients $b_{m,n}$. From the obtained values of abscissas it is necessary to exclude those that are located outside the range of $\omega_N \div \omega_m$ and are "false". To do this, we introduce the inclusion CPLF

$$Q_{m,n}(\varrho) = K_\sigma \sum_{\lambda=0}^1 \sum_{\gamma=0}^1 (-1)^{\lambda+\gamma} \left| \varrho + \varrho_n - \varrho_m(1-\gamma) - \frac{\lambda}{2K_\sigma} \right|$$

where K_σ is a slope of the lateral components of inclusion function. The function takes a value of 1 if its argument takes a segment $[\omega_n; \omega_m]$ and 0 otherwise.

The corresponding inclusion function for "false" abscissas values is equal to zero, and for the true values $Q_{m,n}(b_{m,n}) = 1$. To exclude "false" $b_{m,n}$ values, it is enough to multiply (6) by $Q_{m,n}(b_{m,n})$

$$b_{m,n}^* = b_{m,n} Q_{m,n}(b_{m,n}) \tag{7}$$

We obtain the boundary values of k for each true abscissa by substituting (7) into the right part of (4)

$$k_{m,n} = -1/b_{m,n}^* \tag{8}$$

We denote N_2^{low} and N_2^{up} the lower and upper bounds of the range of N_2 values, in which PSG maintains stability. I.e., the stability region is an interval $N_2^{low} \leq N_2 \leq N_2^{up}$.

In order to find the boundaries of the absolute stability region, it is necessary to select one negative and one positive from all the values (8), which are nearest to zero.

The lower bound of N_2 is defined as the maximum of all negative values

$$\tilde{N}_2^{low} = \max \{ k_{m,n} [1 - \tilde{q}(k_{m,n})] \}, \tag{9}$$

where $\tilde{q}(\varrho) = \frac{1}{2\Delta} [|\varrho + \Delta| - |\varrho - \Delta|]$ - inclusion CPLF, taking value 1 when $\varrho \geq 0$ and 0 when $\varrho < 0$.

The multiplier $1 - \tilde{q}(k_{m,n})$ in (9) excludes positive roots.

The upper bound of N_2 corresponds to the minimum of all $k_{m,n}$ positive values

$$\tilde{N}_2^{up} = \min \{ N_{2k} \tilde{q}(k_{m,n}) \}. \tag{10}$$

As an example, we can calculate the area of absolute stability of generator when low-pass filter (LPF) of the 5th order with transfer function $W(p) = 1/(1 + Tp)^5$ is used as F2, where T is the time constant of the filter. In [15] the investigation of stability "in the small" is made for this filter.

The real and imaginary frequency characteristics of this filter are obtained from the expression (2) based on polynomials $A_{1,2}(\omega)$, $B_{1,2}(\omega)$. We take the time constant of the filter equal to $T=1s$ and approximate the frequency characteristics in the range of variables $\omega_0 = 0,01c^{-1}$, $\omega_N = 7c^{-1}$, $N=30$. Of all the $N_2=900$ abscissas (6) define the true (7). According to (9) $\tilde{N}_2^{low} = -1,006$, to (10) $\tilde{N}_2^{up} = 2,9$.

[Fig. 2] shows the conventional W and the modified W^* hodographs of the frequency characteristic of the filter, approximated by CPLF. Abscissas of approximating lines corresponding to the boundaries of the region of absolute stability, $\tilde{b}_{low} = -1/\tilde{N}_2^{low} = 0,994$, $\tilde{b}_{up} = -1/\tilde{N}_2^{up} = -0,345$. Popov straight lines for $\tilde{N}_2^{low,up}$, indicated in [Fig. 2] as $W_I^{*low,up}$ are obtained by (5).

Stability of the linear generator in accordance with [8, 10] is determined by the condition of intersection of the abscissa axis by the conventional hodograph (W), the boundary values are equal to $N_2^{low} = -1$, $N_2^{up} = 2,885$, that is, stability region of the linear generator with the 5th order LPF in CP^2 represents a segment $-1 \leq N_2 \leq 2,885$.

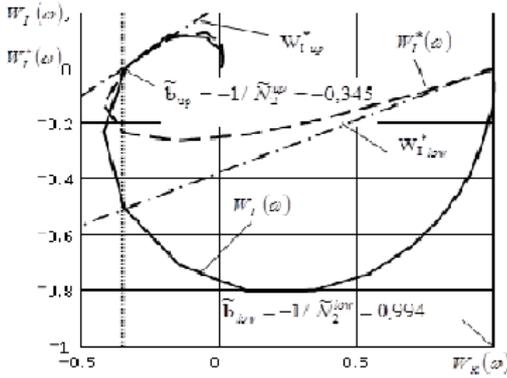


Fig. 2: Conventional W and modified W* hodographs of the 5th order low-pass filter.

From the comparison of the hodographs in [Fig. 2] it is seen that the stability region of the linear PSG and the absolute stability of the nonlinear PSG are the same. A small error of $\tilde{N}_2^{low,up}$ (less than 1%) is caused by an approximation error of frequency characteristics.

Similarly, the stability analysis of PSG with filters of other orders may be done.

In the above case with the 5th order LPF the stability regions of the device in the linear and nonlinear modes coincide. Their difference is also possible for some types of filters. As such an example, we can also calculate the region of absolute stability of the generator with a complex filter of the 4th order in the deviation regulation chain. The filter consists of series-connected integrating, inertia-integrating and oscillatory units. Transfer function of such filter is

$$W(p) = \frac{1}{p(1 + Tp)(\beta + 2\xi Tp + T^2 p^2)}, \tag{11}$$

where T is the time constant of the filter. We assume $T=\xi=1, \beta=10$.

Calculating filter coefficients α, β_i , we obtain the real and imaginary frequency characteristic by substitution of these coefficients in (2). Approximation of the frequency characteristics is feasible in the same range of variables $\omega_0 = 0,01c^{-1}, \omega_N = 7c^{-1}$, as for the 5th order LPF, the number of approximation nodes is increased to $N=50$ to improve accuracy. Of all $N_2=2500$ abscissas (6), let us determine the true ones (7). According to (9) $\tilde{N}_2^{low} = -1/\infty \rightarrow 0$. Upper absolute sustainable N_2 value should be received by (10): $\tilde{N}_2^{up} = 1/0,045 = 22,2$. The region of absolute stability is the interval $0 \leq N_2 \leq 22,2$.

From a comparison of the conventional and modified hodographs in [Fig. 3] it can be seen that the stable region in the linear and nonlinear regimes differ. Thus, stability "in the small" is determined by abscissa of the point at which $W_I(\omega) = 0 : b_{up} = -0,034, N_2^{up} = 1/0,034 = 29,4$. The stability region "in small" is a segment $0 \leq N_2 \leq 29,4$. General method for analyzing the stability of the device in a linear mode (with small perturbations) based on the Nyquist frequency criterion and piecewise linear approximation of the hodograph is described in [15].

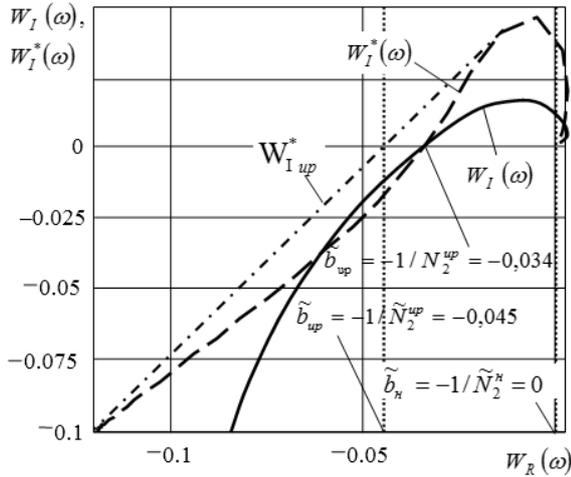


Fig. 3: Conventional and modified hodographs of the 4th order complex oscillation-integrating link.

The upper limit value of N_2 in the nonlinear mode is significantly less than in the linear mode (1.32 times). This imposes restrictions on the choice of parameters of feedback devices for large acting disturbances.

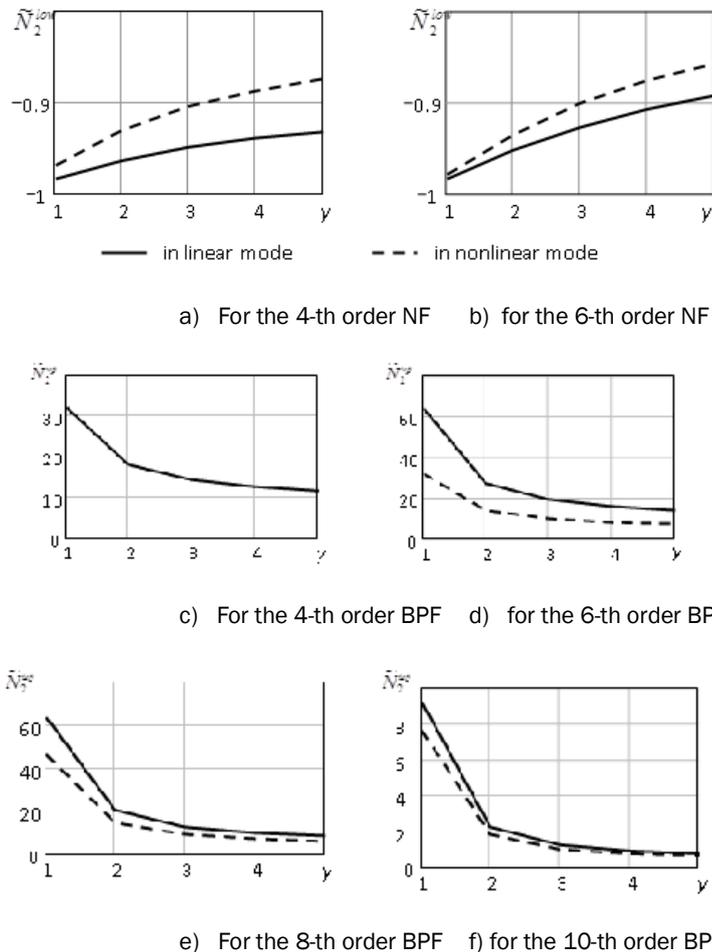


Fig. 4: Dependences of the boundary stable coefficients N_2 on γ in the linear and nonlinear regime for different types and orders of the filter.

In [Fig. 4] graphs of N_2 stable coefficients of generator with band pass (BPF) and notch (NF) filters of different orders (4, 6, 8, 10th) are also given. Transfer functions of filters have the form:

$$M_2^{LPF}(p) = 1/(1+Tp)^l, \quad M_2^{HPF}(p) = (Tp)^l/(1+Tp)^l,$$

$$M_2^{BPF}(p) = H_{LPF}(p)H_{HPF}(p) = (\gamma Tp)^{0.5l} / [(1+Tp)(1+\gamma Tp)]^{0.5l},$$

$$M_2^{\text{NF}}(p) = 1 - M_2^{\text{BPF}}(p).$$

Here T – time constant of the link in the composition of low pass and high pass filter, γ is the ratio of time constants of HPF and LPF links in the part of the BPF and NF. When the de-vice is switched to nonlinear mode of operation, the stability area is significantly narrowed from above (2 times for the 6th order BPF at $\gamma=1$ [Fig. 4d]: $N_2^{\text{up}} = 64$ in linear mode, $\tilde{N}_2^{\text{up}} = 32$ in nonlinear mode). The results of calculation of the lower limit of absolute stability for BPF coincide with the results of calculation for the linear regime. Dependences of absolutely stable N_2 values of the generator with low-pass (LPF) and high-pass (HPF) filters coincide with values for stability "in small" for any order of the filter to the 10th inclusive.

Expressions are obtained that determine the limits of the range of values of the coefficient of deviation corresponding to the stable operation of PSG "in general" (with large values of effects). Application of PSG and CPLF allows to investigate the absolute stability of devices with different type and order of filter in the feedback circuit.

CONCLUSION

The urgency of the study of absolute stability of probing signal generators to ensure the reliability of the phase-metric systems of geodynamic control at an arbitrary value of the influencing conditions (stability "in general") is noted. The method of the absolute stability analysis of high-order generators with different types of filters of control paths is developed. New approach is based on the Popov frequency criterion and the piecewise-linear approximation of the hodograph. The choice of a specific filter of the control path is carried out by simple substitution of its coefficients in the obtained expressions of the generalized PSG model. Computational experiment was conducted to analyze the stability of the PSG with different types and orders of filters: low-pass filters, high-frequency filters, band pass and notch filters from the 1st to the 10th order. Studies had shown a significant difference calculated at the boundary regulation coefficients of generator in linear and nonlinear regime.

CONFLICT OF INTEREST

There is no conflict of interest.

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None

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